“I’m Just Not Good At Math” – Why Do We Tolerate This Statement?

“While it is true that some people are better at math than others,” says University of Virginia cognitive psychologist Daniel Willingham in this American Educator column, “– just like some are better than others at writing or building cabinets or anything else – it is also true that the vast majority of people are fully capable of learning K-12 mathematics.” What level does Willingham have in mind? High-school algebra, geometry, and probability and how they apply to our daily lives.

To what degree are our brains wired to learn math? It doesn’t come as naturally as speech; like learning to read, says Willingham, learning math “takes time and effort, and requires mastering increasingly complex skills and content.” Two important findings of the last 20 years provide the base on which teachers can build:

• Humans are born with the ability to appreciate the concept of numbers. We can quickly approximate how many objects are in front of us, for example, being able to tell at a glance that 100 beans scattered on one table are more than 50 beans scattered on another. Children can also gauge exact numbers up to four with precision; their error rate increases after that until they master counting.

• Humans seem to have an innate sense that numbers and space are related – number lines are used independently in many different cultures – but precise one-to-one correspondence of numbers greater than four on a number line takes teaching.

What’s the bottom line from cognitive science? Willingham says “we should not expect students will learn mathematics with ease. Rather, we should expect that mathematical proficiency will require careful cultivation and will develop slowly. At the same time, we should keep in mind that students are born with the ability to learn math, and we should not let students give up by concluding that they’re just not good at math.”

So what do students need if they are to be successful at math? The National Mathematics Advisory Panel (NMAP) recently zeroed in on three things:

- Factual knowledge – Memorizing the answers to a relatively small set of addition, subtraction, multiplication and division problems so that retrieval is quick and automatic. This is critical to solving more complex problems because it frees up working memory to focus on higher-order thinking.

- Procedural knowledge – Learning sequences of steps to solve frequently encountered problems, for example, borrow-and-regroup or knowing that multiplying two negative numbers produces a positive number (not why).

- Conceptual knowledge – It’s not either-or between procedural and conceptual knowledge, as protagonists in the math wars have contended – children need both. But does one come before the other – concepts before procedures or procedures before concepts? The answer, says Willingham, is to teach both together!
How well are American students doing in these three areas? Willingham believes our kids’ grasp of factual and procedural knowledge is middling and their great weakness is conceptual knowledge. Many students don’t fully understand place value, which led one college freshman to argue strenuously that 0.015 is larger than 0.05 because 15 is larger than 5. She would not budge from this belief. Another common misconception is believing that the equal sign = means “put the answer here” rather than denoting mathematical equivalence.

Unfortunately, says Willingham, conceptual knowledge is the hardest of the three to acquire, and it can’t just be poured into students’ heads; it needs to be built on previous knowledge and experience. For example, an abstract definition – the standard deviation is a measure of the dispersion of a distribution – is much easier to understand with an example – two groups of people have the same average height, but one group has many tall and many short people, and thus has a large standard deviation, whereas the other group mostly has people right around the average, and thus has a small standard deviation. “If students fail to gain conceptual understanding,” says Willingham, “it will become harder and harder to catch up, as new conceptual knowledge depends on the old. Students will become more and more likely to simply memorize algorithms and apply them without understanding.”

Manipulatives are touted as a way to help students gain conceptual understanding, but Willingham is skeptical. Manipulatives are themselves abstract, he says, and can be so visually interesting that they distract from the deeper concepts involved. “Concreteness is not a magical property that allows teachers to pour content into students’ minds,” he says. Working with familiar objects or examples is more important to helping students understand abstract ideas. “Crucially, students frequently fail to understand the concept if they are not explicitly told to look for the commonalities among examples, or are not given hints as to what the commonalities are.” As content becomes more abstract, teachers have to be very explicit with their examples and analogies, for example, using a balance scale to illustrate the balancing of two sides of an equation, making it vivid by actually having a balance scale in the classroom and showing how it works, making the analogy explicit by writing two sides of an equation on a drawing of a balance scale, and continuously reinforcing the analogy. Studies indicate that teachers in Japan and Hong Kong are especially effective at using analogies according to these principles.

What are the implications of this for American teachers? Willingham offers the following advice:

• Think carefully about how to cultivate conceptual knowledge, and find an analogy that can be used across topics so that students will make connections.

• In cultivating greater conceptual knowledge, don’t sacrifice the basics. “Procedural or factual knowledge without conceptual knowledge is shallow and is unlikely to transfer to new concepts,” he says, “but conceptual knowledge without procedural or factual knowledge is ineffectual.” The “how” and “why” of math reinforce each other.

• In teaching procedural and factual knowledge, ensure that students get to automaticity.
• Choose a curriculum that supports conceptual knowledge – which means a curriculum that focuses on just a few concepts each year and a sequence of topics that makes conceptual sense and proceeds in small steps.

• Don’t tolerate the statement, “I’m just no good at math” from a student. With persistence and hard work, students can do it!

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